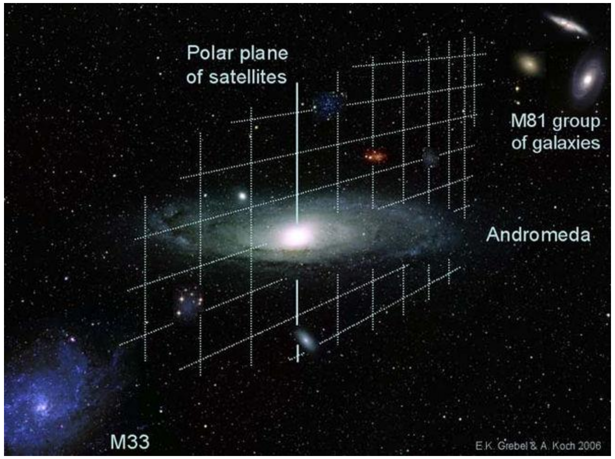
Constraining the tangential velocities of Andromeda satellites using non linear optimization



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September 30 2016

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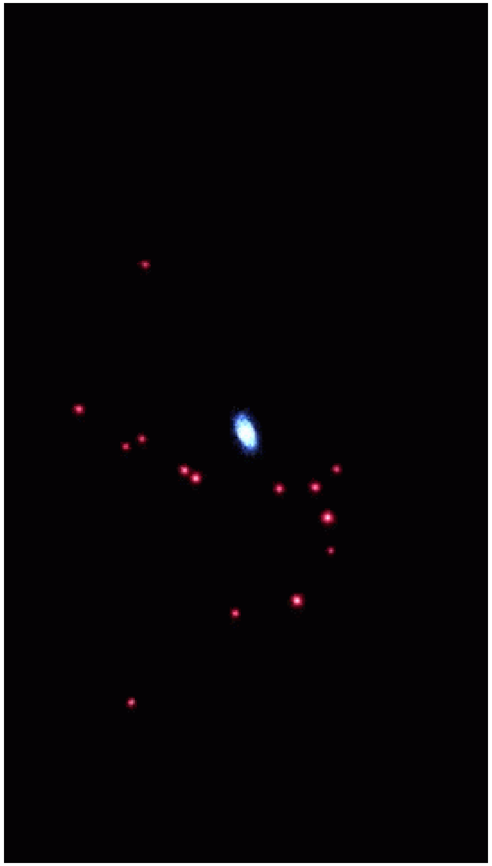
Content

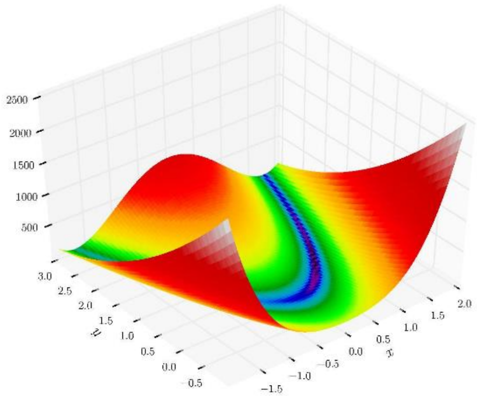
Introduction

- Ibata et al. (2013)[1] proposed the presence of a vast thin plane of corrotating dwarf galaxies or- biting the Andromeda Galaxy.
- Dynamical simulations of system have been made using observational data, in order to propose the time evolution of this structure.
- Tangential velocity of the satellite galaxies had to be supposed
- Unavailable observational data for this magnitudes.



e. a. Conn, «The Three-Dimensional Structure Of The M31 Satellite System; Strong Evidence For An Inhomogeneous Distribution Of Satellites,» Astrophysical Journal, 2013.

Introduction

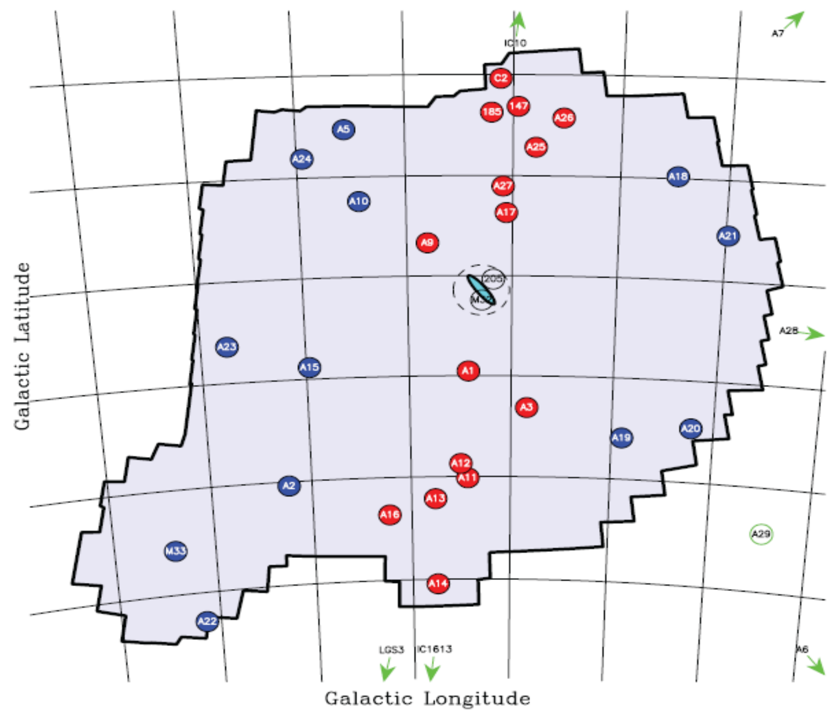


https://raw.githubusercontent.com/OSGConnect/tutorial-matlab-SimulatedAnnealing/master/Figs/RosenBrockFunction.png

- Non Linear Optimization Algorithms (NLP) of large scale to constrain the values of this tangential velocities
- Minimize the difference between the different initial conditions and trajectories of the dwarf galaxies, and the simulated state of the system, constrained to the current observed state.
- shed light on the possible formation processes and stability of this structure of galaxies in our local cluster.

General Objective

Use large scale Non Linear Programming (NLP) optimization algorithms to determine the values the tangential velocities of Andromeda satellites, according to different initial conditions.



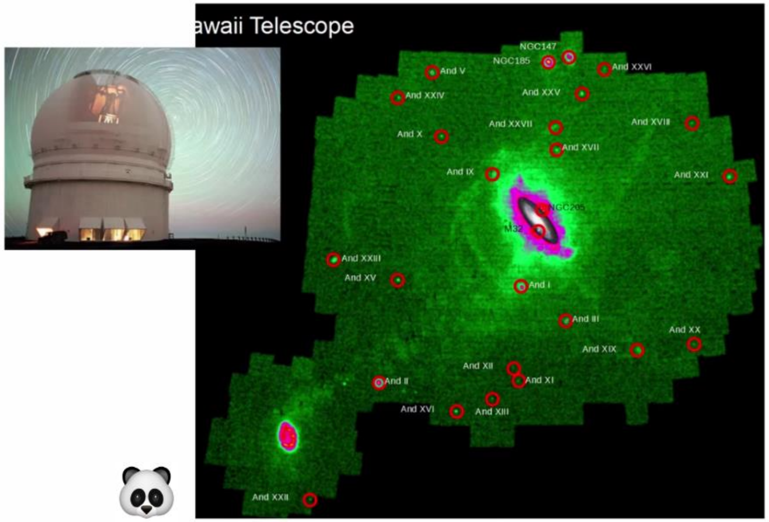
e. a. Ibata, «A Vast Thin Plane of Co-rotating Dwarf Galaxies Orbiting the Andromeda Galaxy,» 2013.

Specific Objectives

- Simulate the dynamic behavior of the Andromeda system and it's satellite galaxies.
- Implement the set of equations describing the system in the software *General Algebraic Modeling System* (GAMS)[2] to apply NLP optimizations algorithms.
- Propose a set of initial conditions and trajectories to evaluate.
- Solve the numerical optimization of the minimizing problem.
- Determine the feasibility of the initial conditions proposed based on the results of the numerical optimization.

Methodology

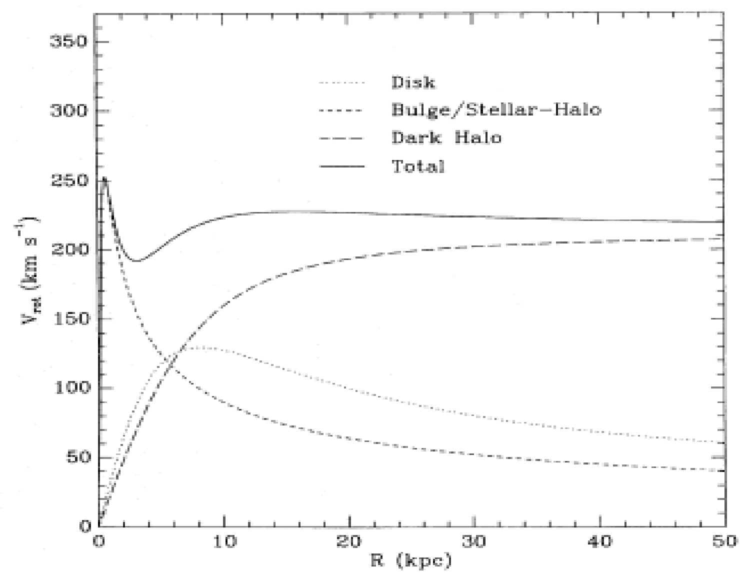
- Obtain information from The Pan-Andromeda Archaeological Survey (PAndAS)[3] referent to the mass, redshift, position and other parameters.
- Obtain parameters from other sources (Ibata et al.[1], Conn et al.[4] y Koch et al. [5])



Pandas' progeny: extending the m31 dwarf galaxy cabal, Richardson et al.

Methodology

- Andromeda and Milky way's exerted gravitational potential is composed by 3 parts.
- Modeling of the gravitational interaction with these two galaxies has been obtained from Hernquist[6] y Miyamoto y Nagai[7] work.

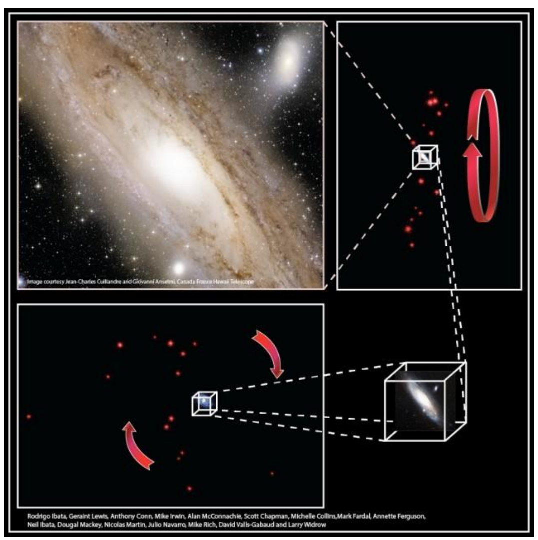


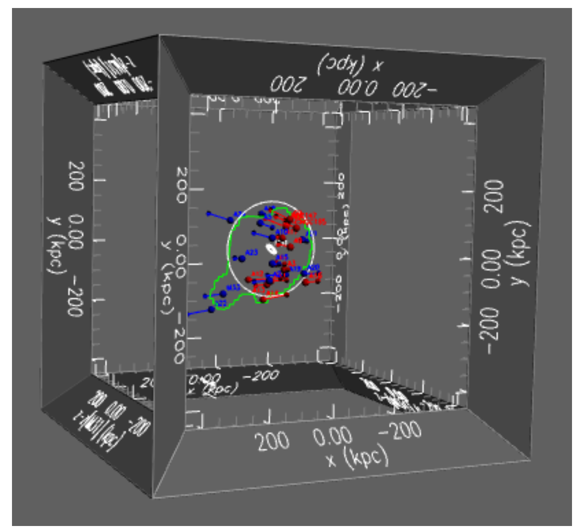
C. Flynn, J. Sommer-Larsen, and P. R. Christensen, "Kinematics of the outer stellar halo," *Mon. Not. R. Astron. Soc.*, vol. 281, p. 1027, 1996.

Methodology

- Sympletic integrator to assure the energy conservation within the system, as an example Flynn et al.[8]

- Simulations with 5Gy in the past and future.





Problem Statement

Sets:

S: Satellites, index i

$S_p \subseteq S$: Satellites in the plane, index i

$S_{np} \subseteq S$: Satellites not in the plane, index i

$T = \{0, \ldots, \pm 1Gy\}$: Time, index j

***Parameters and scalars**:*

Obtained by experiments and authors.

Variables:

$V_x^{tan}(i); V_y^{tan}(i);\ V_z^{tan}(i); \forall i \in S$:

$x(i,j); y(i,j); z(i,j); \forall i \in S; \forall j \in T$:

Problem Statement

***Constraints**:*

All the equations defining the system, and variable delimitations.

Objective Function:

Here is a query. ¿What do we want to Maximize/Minimize? ¿How do we want the tangential velocities to take effect over the system?

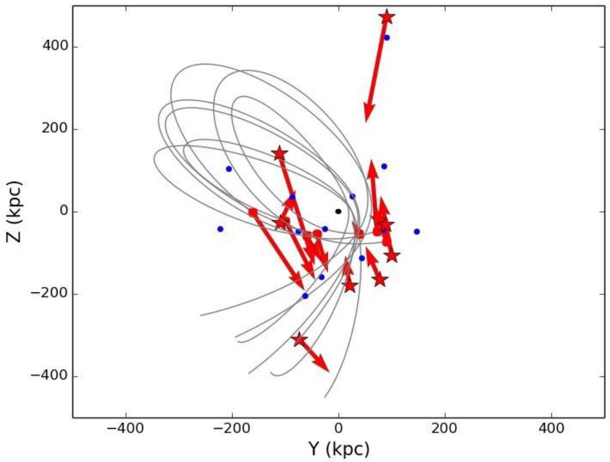
Initially all together.

$$\min \sum_{j\in T} \sum_{i\in S_p} \sum_{k\in S_p} \left(\left(x(i,j) - x_0(k,0)\right)^2 + \left(y(i,j) - y_0(k,0)\right)^2 + \left(z(i,j) - z_0(k,0)\right)^2 \right)$$

Dynamic simulation of Andromeda satellites and galaxies

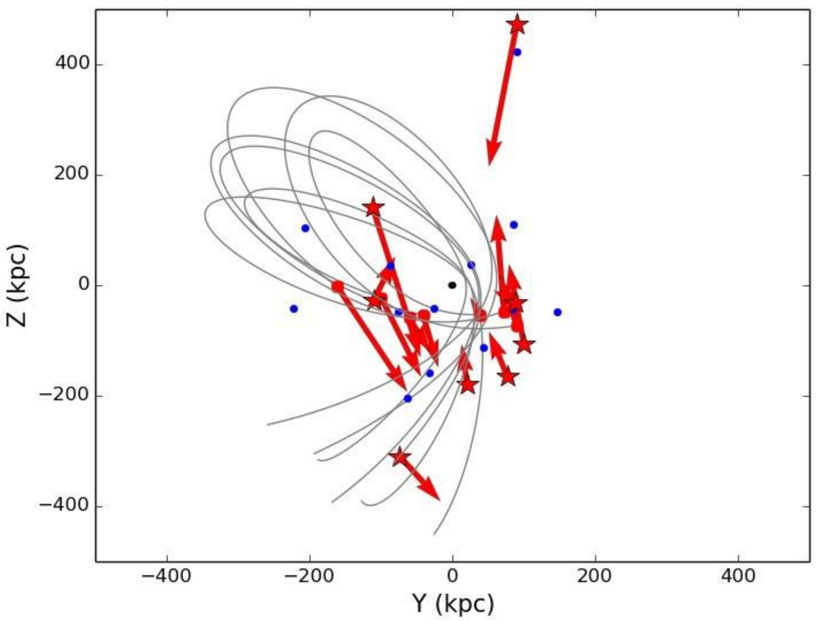
First part of this work was done by Verónica Arias (PostDoc investigator).

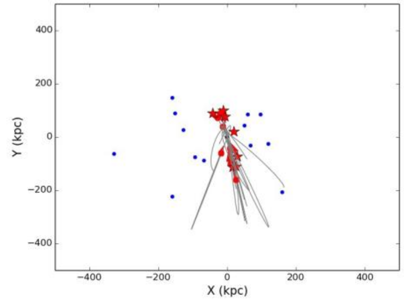
- Assumed all satellites within the plane, would be confined to it.
- This assumption reduces the degrees of freedom to one per satellite.
- By a manual iterative method, she found a set of velocities that made 7 out of 15 satellites in the plane follow a similar trajectory over the others.

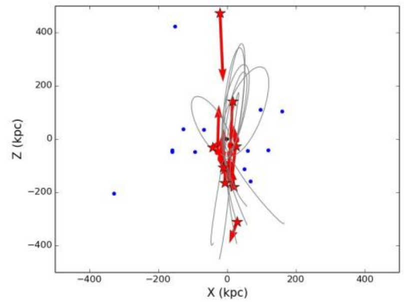


Dynamic simulation of Andromeda satellites and galaxies

Using the same tangential velocities used by Verónica, the GAMS implementation achieves the same results.







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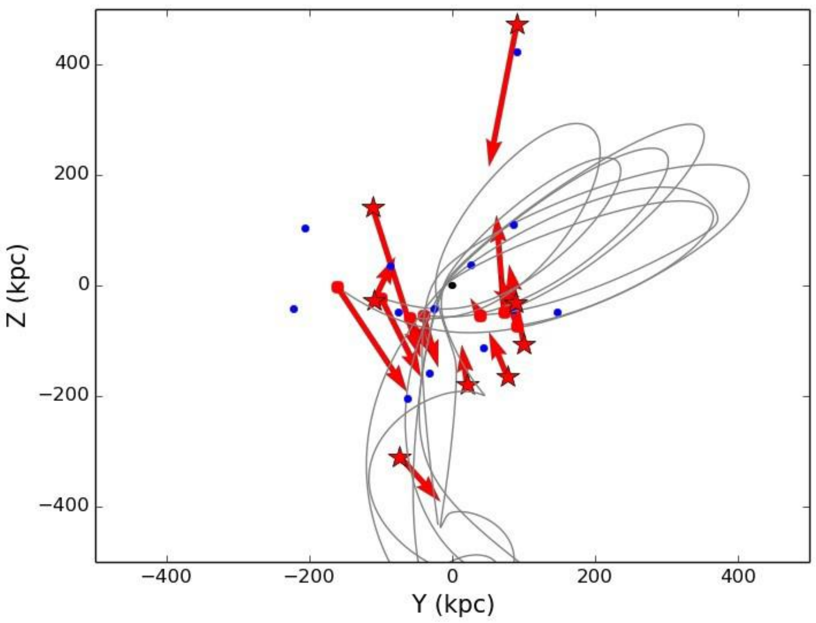

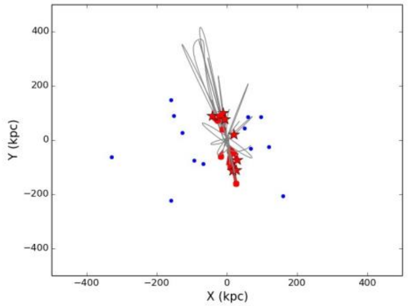

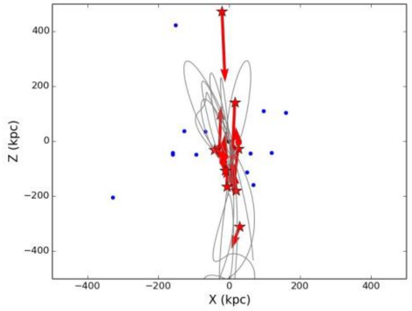

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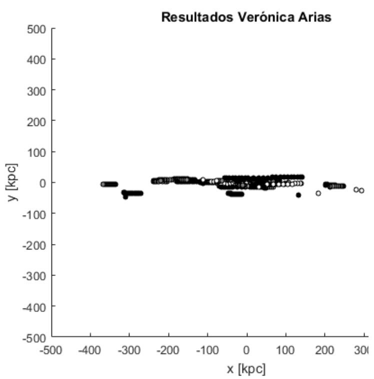

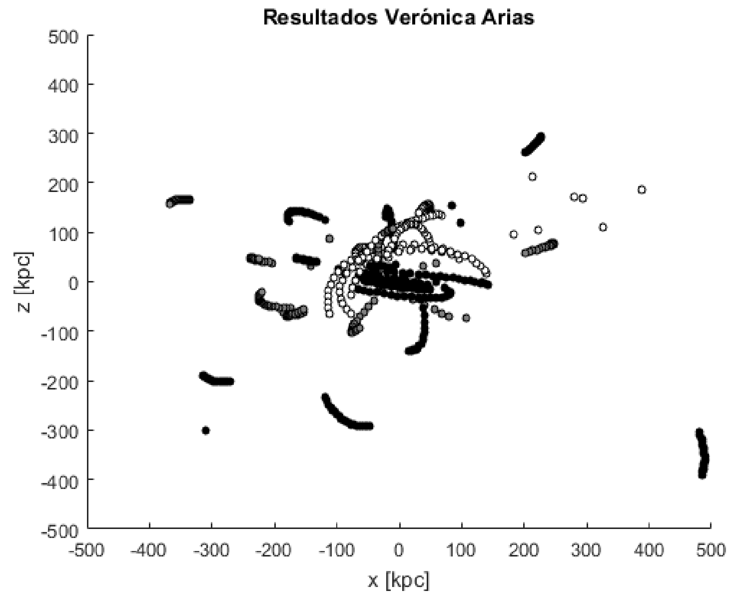

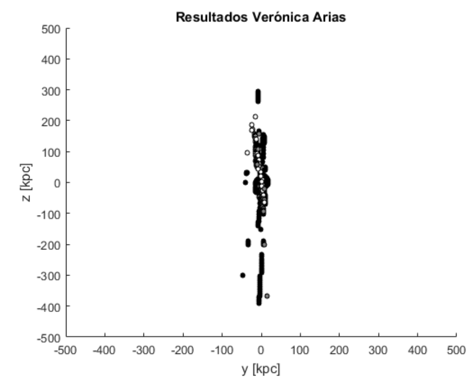

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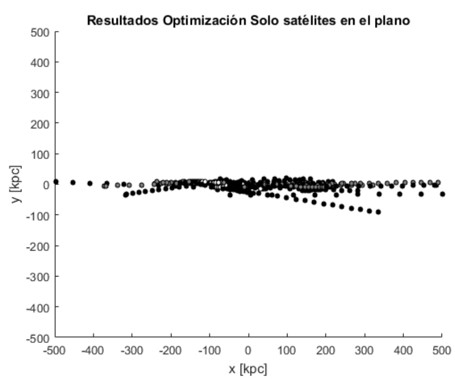

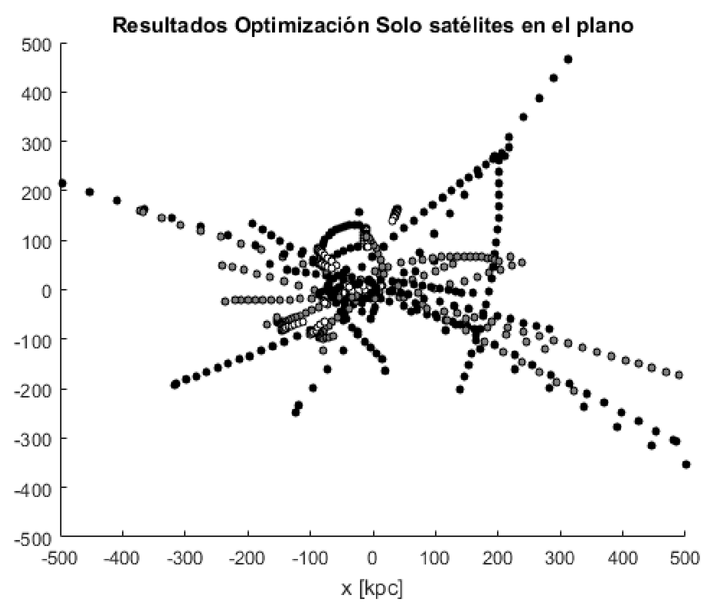

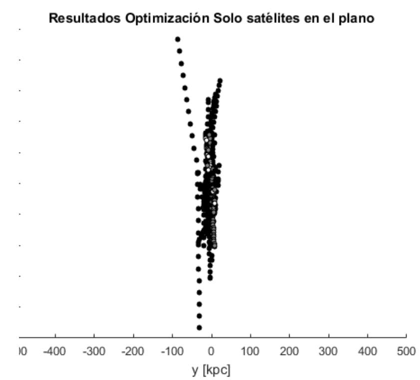

Introduction | Objectives | Methodology | Problem Statement | Dynamic simulation of Andromeda satellites and galaxies | Scope of the project

Dynamic simulation of Andromeda satellites and galaxies

Using the same tangential velocities used by Verónica, the GAMS implementation achieves the same results.

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Space for names approximately 13.07 MB.
Use statement '<modelname>.dictfile=0;' to turn dictionary off.
Number of nonzeros in equality constraint Jacobian...:  3562440
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian.............:  1696940

Total number of variables............................:   213360
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:   213192
Total number of inequality constraints...............:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0
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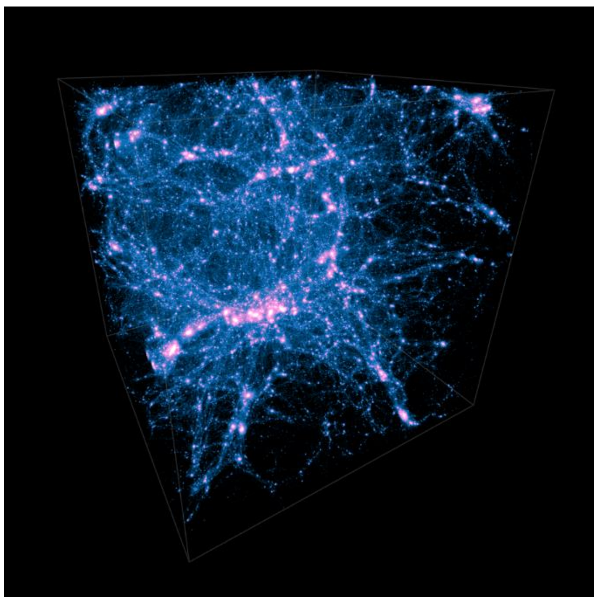
Dynamic simulation of Andromeda satellites and galaxies

Velocities will be obtained initially in a position dot product velocity distribution obtained of 896 objects of Illustris simulation.



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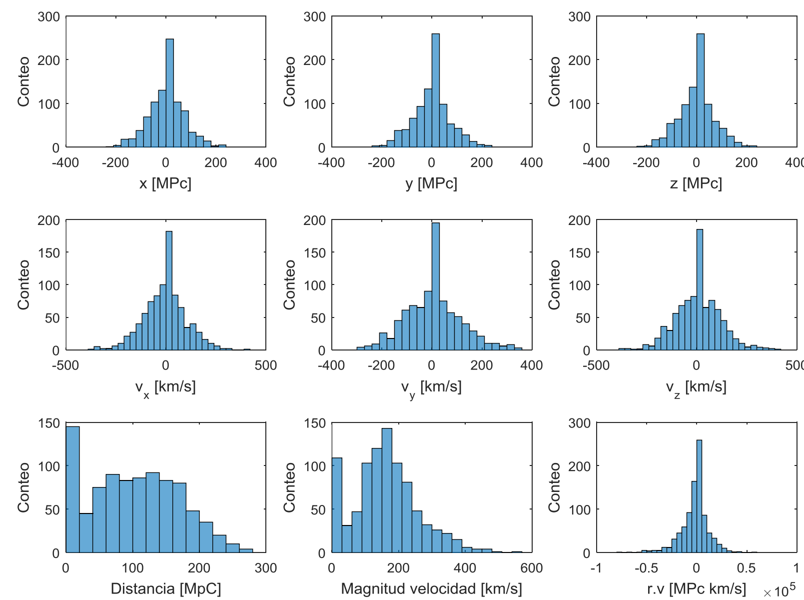

Scope of the project

Current scope and preliminary conclusions:

- The definition of an optimization problem was achieved, to vary tangential velocities of Andromeda satellites for it's dynamic behavior.

- The implementation and simulation of the dynamic model of Andromeda satellites was achieved on Python and GAMS.

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Scope of the project

Future Work:

- Obtain the parameters of tangential velocities through non linear optimization on GAMS.
- Propose different objective functions associated to the satellite conditions.
- Analyze the stability and feasibility of the obtained results.

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Referencias

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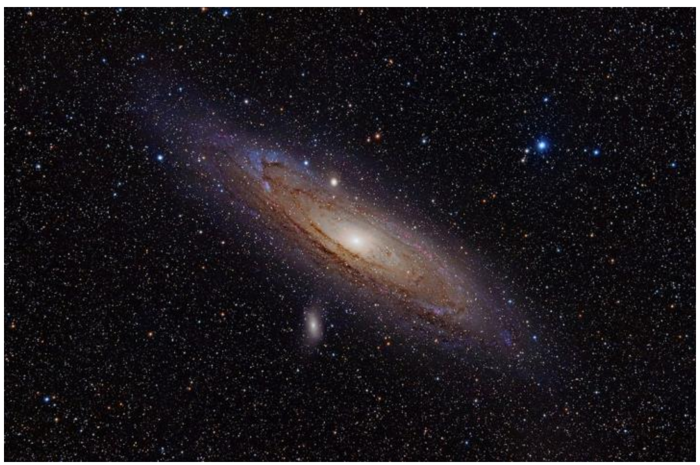

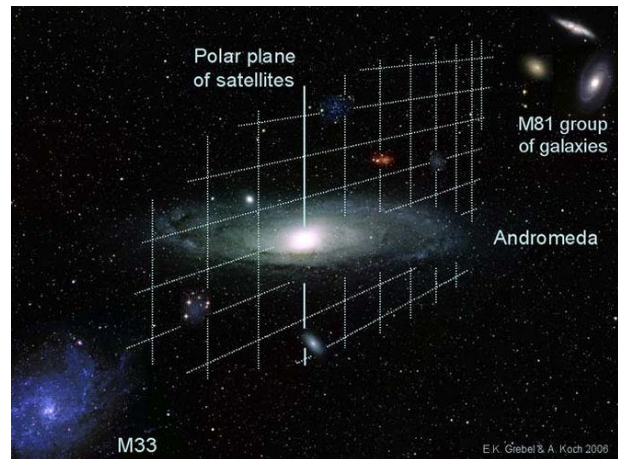

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Gravitational Potential

Andromeda potential

$$\Phi_{bulge}(r) + \Phi_{disk}(r) + \Phi_{halo}(r)$$
$$= -\frac{GM_{bulge}}{r_{bulge} + r} - -2\pi\Sigma_0 r_{disk}^2 \left[\frac{1 - e^{-\frac{r}{r_{disk}}}}{r}\right] - \frac{GM_{halo}}{r}\log\left(\frac{r}{r_{halo}} + 1\right)$$

Milky way potential

$$\Phi_{bulge}(r) + \Phi_{disk}(r) + \Phi_{halo}(r)$$
$$= -\frac{GM_{bulge}}{r_{bulge} + r} - \frac{GM_{disk}}{\sqrt{R^2 + \left(r_{disk} + \sqrt{z^2 + b^2}\right)^2}} - \frac{GM_{halo}}{r}\log\left(\frac{r}{r_{halo}} + 1\right)$$

Detailed Problem Statement

Sets:

S: Satellites, index i

$S_p \subseteq S$: Satellites in the plane, index i

$S_{np} \subseteq S$: Satellites not in the plane, index i

$T = \{0, \ldots, \pm 5Gy\}$: Time, index j

***Parameters and scalars**:*

Obtained by experiments and authors. (In vision line velocities, relative positions, potential characteristics,…)

Variables:

$V_x^{tan}(i); V_y^{tan}(i); V_z^{tan}(i); \forall i \in S$:

Tangential velocities components for all satellites (Only 2 components are independent, since there is information about the non tangential).

$x(i,j); y(i,j); z(i,j); \forall i \in S; \forall j \in T$:

Position of each satellite through time.

Detailed Problem Statement

Constraints:

All the equations defining the system (movement equations, gravitational potentials, coordinate conversion), and variable delimitations (Not to fast, for the in plane they are restricted to it,…)

Objective function:

Here is a query. ¿What do we want to Maximize/Minimize? ¿How do we want the tangential velocities to take effect over the system?

Initially all together.

$$\min \sum_{j\in T} \sum_{i\in S_p} \sum_{k\in S_p} \left(\left(x(i,j) - x_0(k,0)\right)^2 + \left(y(i,j) - y_0(k,0)\right)^2 + \left(z(i,j) - z_0(k,0)\right)^2 \right)$$