

Particle Filter-based Estimation of Orbital Parameters of Visual Binary Stars with Incomplete Observations

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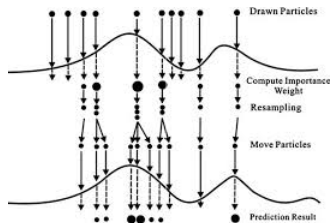
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Introduction

- ▶ Binary (and multiple) stars constitute the observational base for the study of a number of astronomical phenomena (stellar formation, evolution, etc.).
- ▶ Partial measurements: observations of relative position where one component is missing (ρ or θ) or confined to a interval of plausible values instead of a single point (e.g., $\rho \in (0, \rho_{max}]$).
- ▶ The Bayesian approach enables us to combine several sources of information (empirical, statistical, theoretical) when estimating an orbit.
- ▶ An imputation scheme is proposed to incorporate partial knowledge in the analysis.

Particle Filter



- ▶ Particle Filter is based on the representation of p.d.f. as a set of weighted samples, called *particles* $(x_t^{(i)}, w_t^{(i)})$.
- ▶ Weights depend on a likelihood function used to evaluate each sample. Particles are upgraded each time a new measurement is available.
- ▶ In this work, each particle is a set of orbital parameters:

$$x_t^{(i)} = (T_t^{(i)}, P_t^{(i)}, e_t^{(i)}, a_t^{(i)}, \omega_t^{(i)}, \Omega_t^{(i)}, i_t^{(i)}).$$

- ▶ Partial measurements are incorporated by replacing them by a set of plausible values (multiple imputations), in order to generate complete data sets where the PF method can be applied.

Results

Experiment description

- ▶ Parameters of binary star Sirius were used to generate synthetic data, with observation noise having standard deviation $\sigma = 0.075$ [arcsec] for both X and Y axes
- ▶ Three scenarios:
 - ▶ Full data set is available ($N_{obs} = 11$ observations).
 - ▶ Incomplete observations at epochs τ_{10} (X missing), τ_{11} (Y missing), discarded from the analysis.
 - ▶ Same set of measurements, but Multiple Imputation PF is executed in order to incorporate partial information.
- ▶ 10 executions of the PF algorithm for each scenario (in order to assess the method's performance statistically).

Results

Orbit estimates in three scenarios

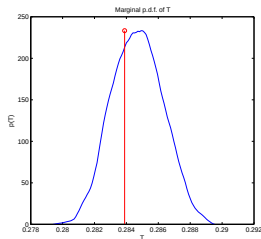
Table: Average value of estimated parameters (std dev in parentheses)

	T	P [yr]	e	a [arcsec]	ω [rad]	Ω [rad]	i [rad]
Real Parameters	0.2839	50.09	0.5923	7.5000	2.5703	0.7779	2.3829
Complete data	0.2820 (0.0033)	50.4178 (0.5571)	0.5961 (0.0045)	7.5030 (0.0289)	2.5639 (0.0234)	0.7735 (0.0201)	2.3766 (0.0075)
Data Discarding	0.2859 (0.0144)	49.8637 (2.3624)	0.5909 (0.0153)	7.4874 (0.1129)	2.5669 (0.0239)	0.7717 (0.0213)	2.3823 (0.0128)
Multiple Imputations	0.2830 (0.0059)	50.2793 (0.9475)	0.5949 (0.0122)	7.5035 (0.0354)	2.5670 (0.0397)	0.7740 (0.0316)	2.3770 (0.0072)

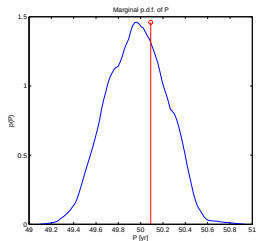
- ▶ In this particular example, none of the three scenarios show significant estimation bias (estimates calculated as weighted average/expected value).
- ▶ However, the incorporation of partial measurements into the analysis contributes to decrease the estimation variance.

Results

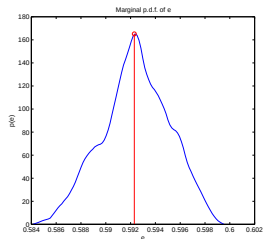
Posterior distribution



Marginal p.d.f. of T



Marginal p.d.f. of P

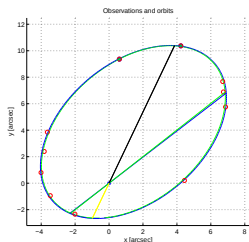


Marginal p.d.f. of e

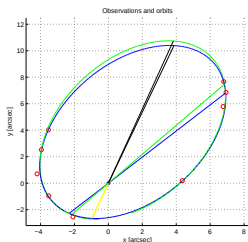
- The figures above show the marginal posteriors obtained with the proposed method in a representative case.

Results

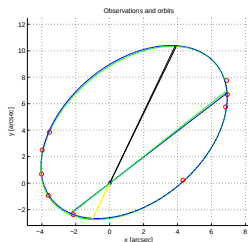
Orbit estimates in three scenarios (visualization)



Complete Data Set



Data Discarding



Imputations

- ▶ The figures display representative cases of the results displayed in the table.
- ▶ When partial information is discarded, orbit estimates (green) show significant discordance with the reference orbit (blue).
- ▶ When partial information is considered, difference between estimated and reference orbit tends to decrease.

Conclusions and Future Work

- ▶ A new particle-filter-based method for estimation of orbital parameters is proposed.
 - ▶ Allows uncertainty characterization. P.d.f. representation can be used to identify multiple solutions instead of choosing only one of them.
 - ▶ Incorporates partial information in the analysis in a formal way.
- ▶ Results suggest that incorporation of partial information could contribute to reduce estimation variance with no appreciable effect on accuracy.
- ▶ Future work:
 - ▶ Testing on real cases.
 - ▶ Using uncertainty characterization in the planning of future observations: deciding when to take measurements and what precision is needed; evaluating what kind of measurement would be more informative (RV, astrometry).